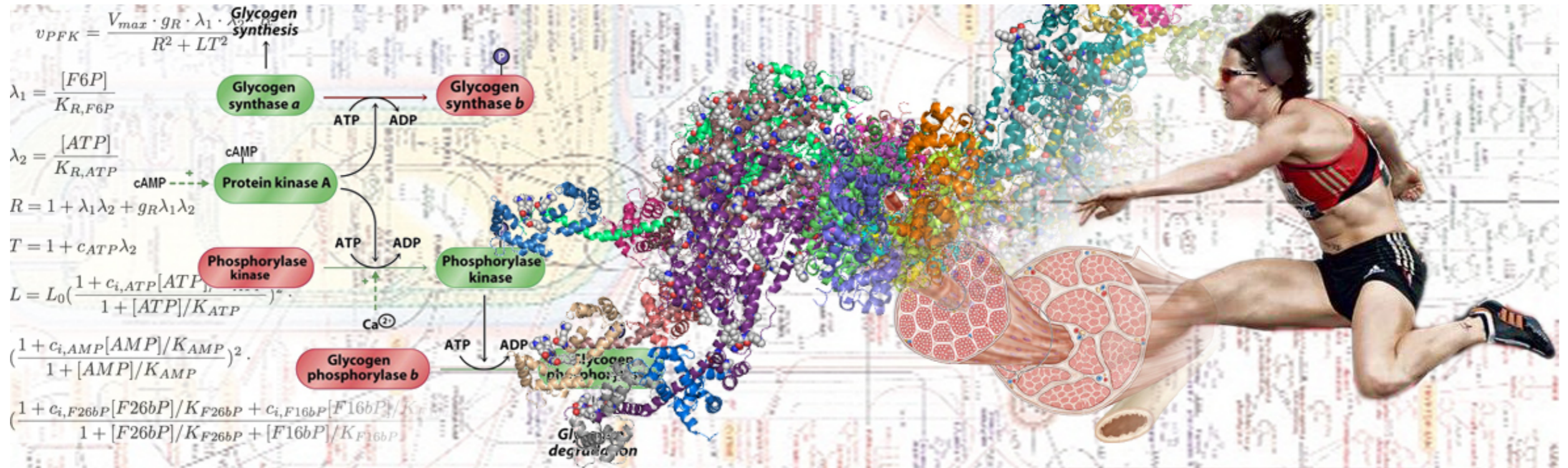


Mini-course: Molecular Systems Biology



Profs Jacky Snoep and Johann Rohwer

March 2018

Thus far

- First Lecture: Chemical kinetics
 - Direction of reaction: dG , ΔG / K_{eq}
 - How far: K_{eq} , dG^0 ; How fast: mass action kinetics
- Second Lecture: Enzyme kinetics
 - Derivation of rate equations: equilibrium binding, steady state approximation
 - V_{max} , K_m , saturation, cooperativity, allostery, reversibility, product inhibition
- Third Lecture: Coupled reactions
 - Parameter estimation; initial rates, progress curves
 - Closed, open systems; equilibrium, steady state, rate characteristics
- Fourth Lecture: Structural network analysis
 - N , K , L matrix
 - Steady state flux constraints, Flux analysis, Flux modes

Exercise

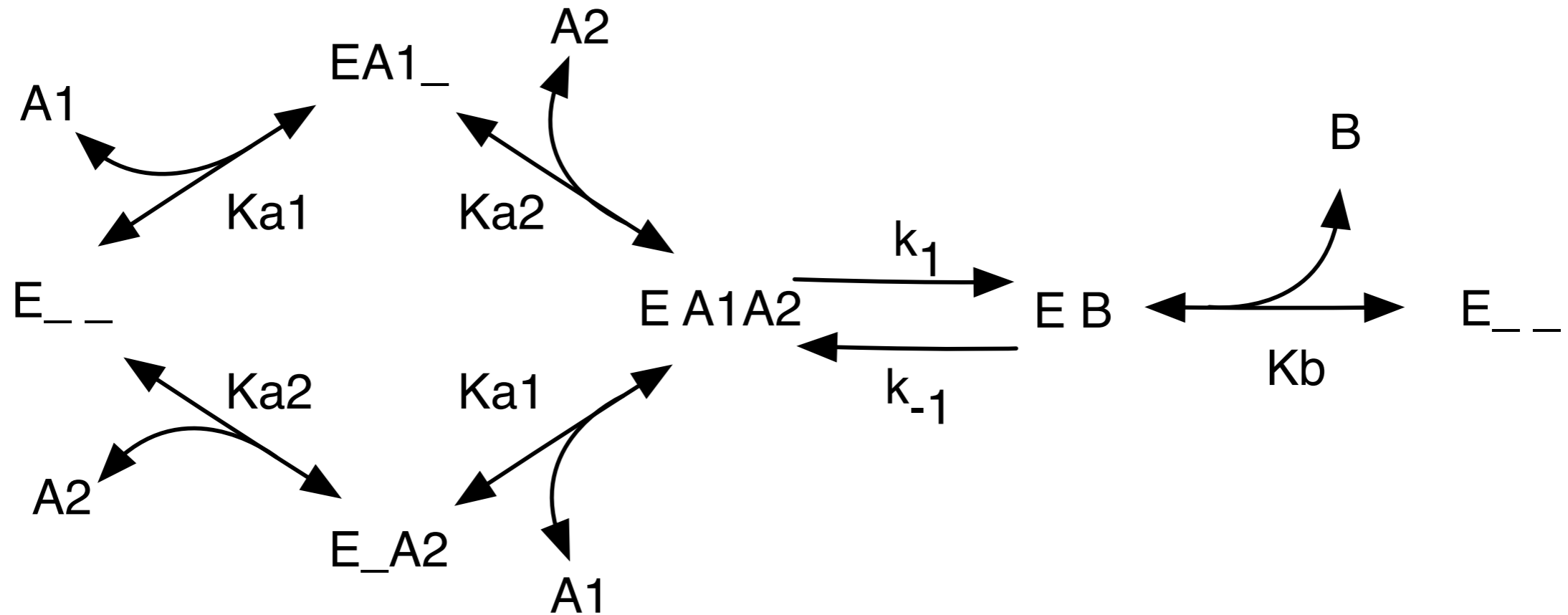
Given an open system consisting of two enzymes that catalyze the conversion of substrate S (fixed at 10 mM) to product P (fixed at 1 mM), with common intermediate X.

The enzymes obey rev MM kinetics with identical parameter values:

$V_m = 1 \text{ mM/s}$, $K_{eq} = 10$, $K_m \text{ substrate} = 1 \text{ mM}$, $K_m \text{ product} = 10 \text{ mM}$

Calculate the steady state flux and the steady state concentration of the intermediate X.

Exercise

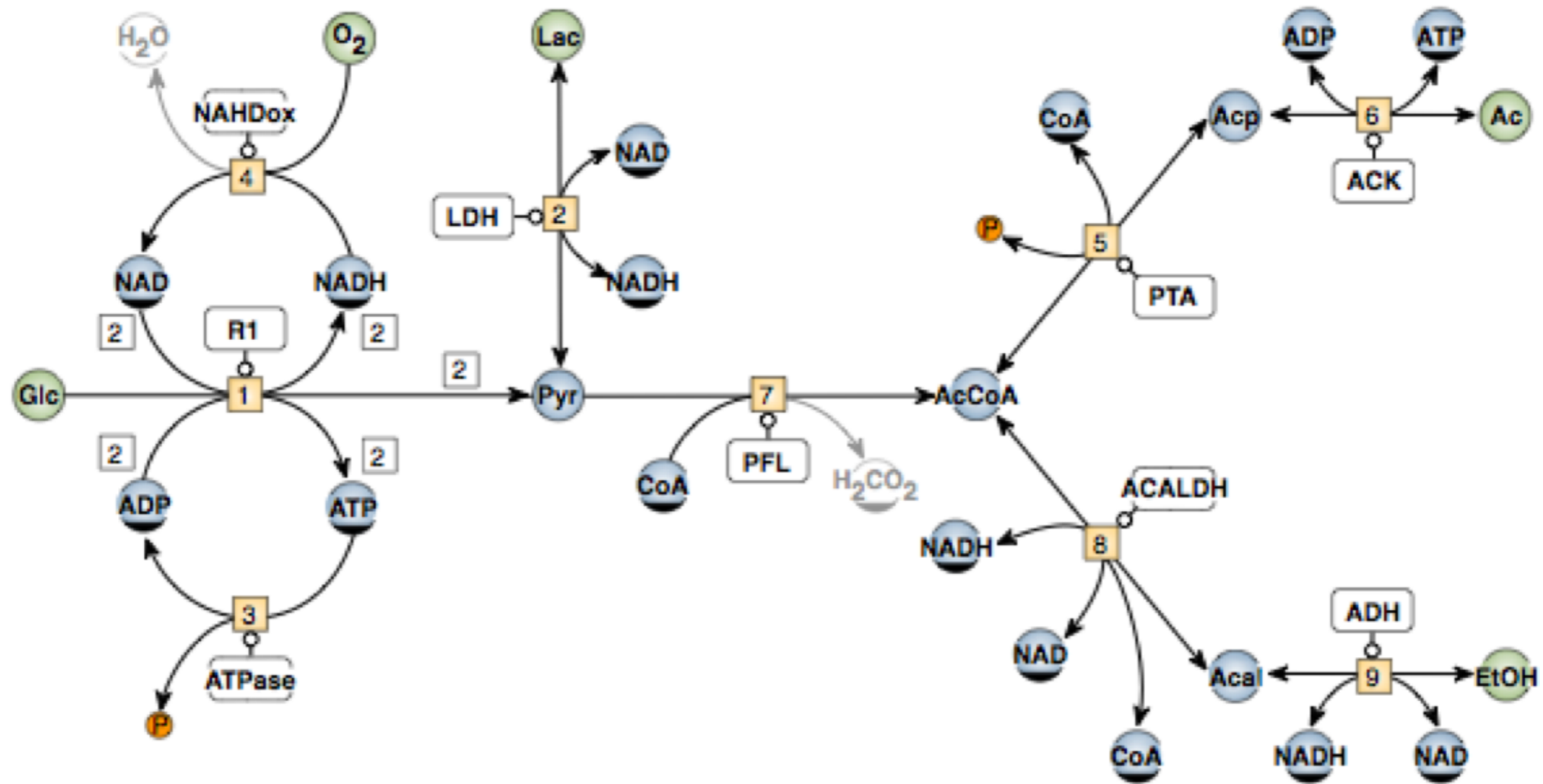


Derive a rate equation for the above, reversible, 2 substrate, 1 product, random order mechanism, using the rapid equilibrium binding assumption.

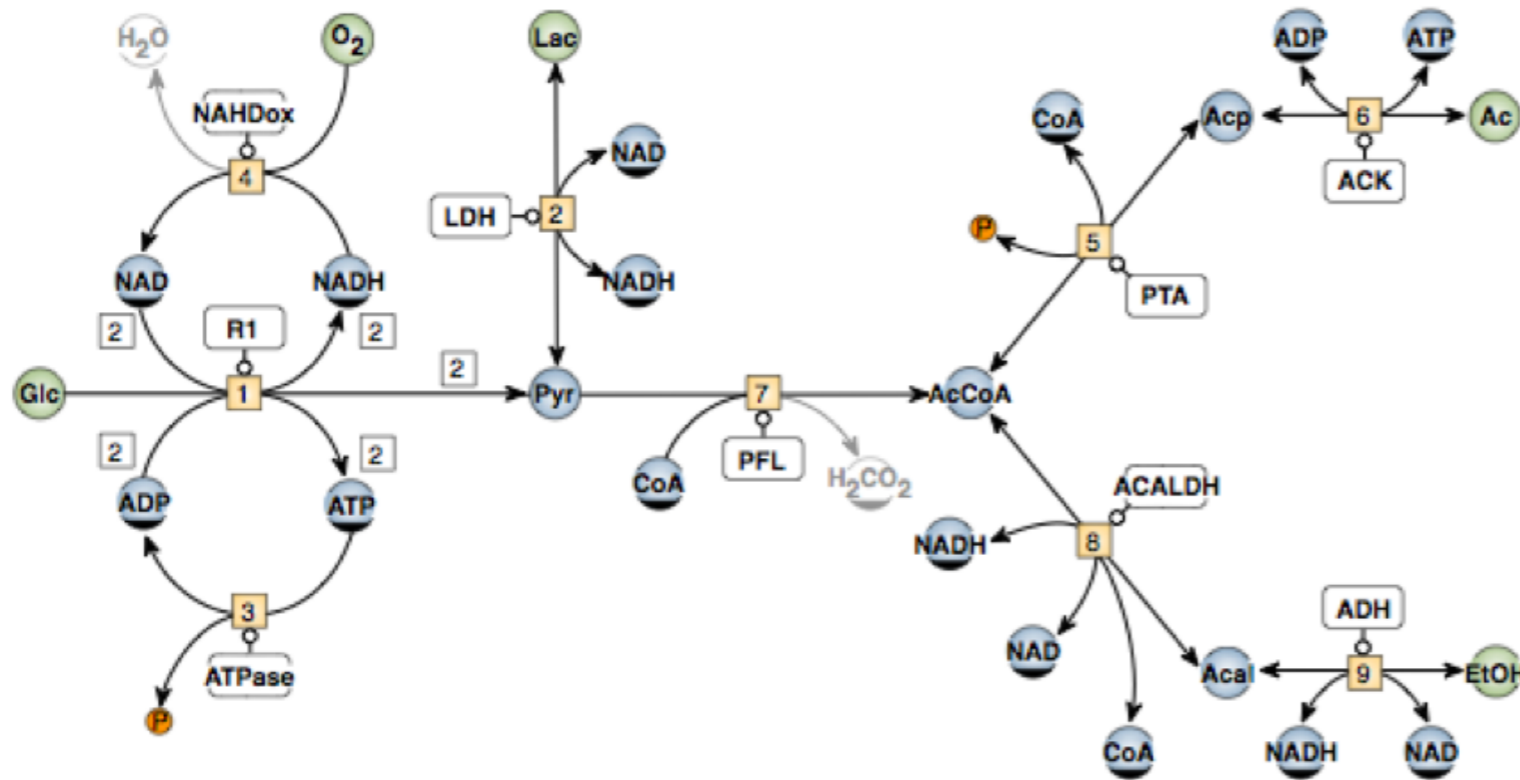
Network analysis

- For a given system, derive N , K , L , matrix, interpret
- Flux modes
- Solution space, FA
- Objective function, FBA

Reaction network



Reaction network: N matrix

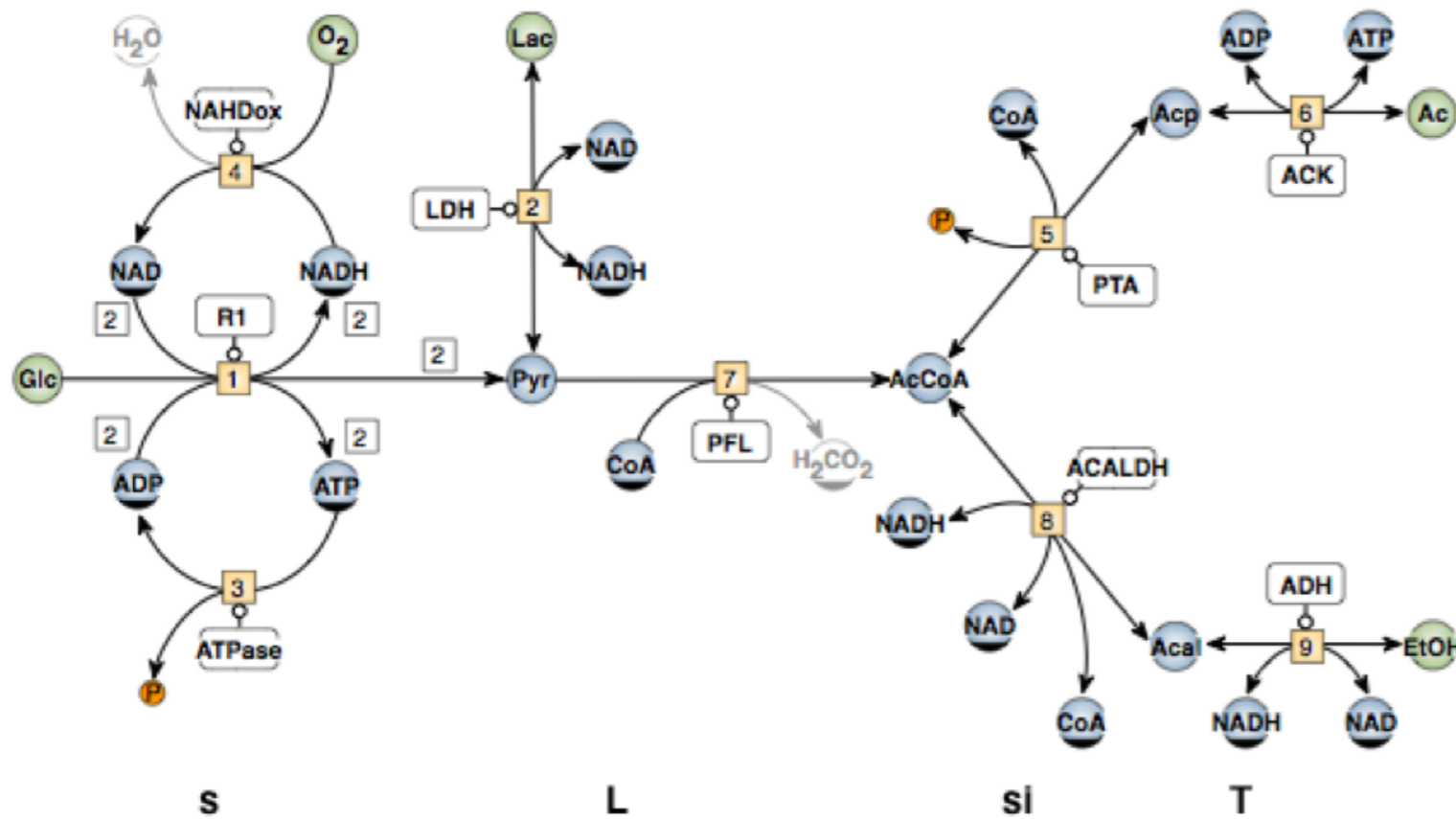


$$\begin{array}{l}
 \frac{d(s)}{dt} \\
 \left(\begin{array}{l}
 d(ACAL)/dt \\
 d(ACCOA)/dt \\
 d(ACP)/dt \\
 d(ADP)/dt \\
 d(ATP)/dt \\
 d(COA)/dt \\
 d(NAD)/dt \\
 d(NADH)/dt \\
 d(PYR)/dt
 \end{array} \right)
 \end{array}
 =
 \begin{array}{c}
 \mathbf{N} \\
 \left(\begin{array}{ccccccccc}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & -1. \\
 0 & 0 & 0 & 0 & -1. & 0 & 1. & -1. & 0 \\
 0 & 0 & 0 & 0 & 1. & -1. & 0 & 0 & 0 \\
 -2. & 0 & 1. & 0 & 0 & -1. & 0 & 0 & 0 \\
 2. & 0 & -1. & 0 & 0 & 1. & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1. & 0 & -1. & 1. & 0 \\
 -2. & 1. & 0 & 1. & 0 & 0 & 0 & 1. & 1. \\
 2. & -1. & 0 & -1. & 0 & 0 & 0 & -1. & -1. \\
 2. & -1. & 0 & 0 & 0 & 0 & -1. & 0 & 0
 \end{array} \right)
 \end{array}
 *
 \begin{array}{l}
 \mathbf{v} \\
 \left(\begin{array}{l}
 v1 \\
 v2 \\
 v3 \\
 v4 \\
 v5 \\
 v6 \\
 v7 \\
 v8 \\
 v9
 \end{array} \right)
 \end{array}$$

N stoichiometry matrix

v reaction rates

Reaction network: L matrix



$$\begin{pmatrix} \text{ACAL} \\ \text{ACCOA} \\ \text{ACP} \\ \text{ADP} \\ \text{NAD} \\ \text{PYR} \\ \text{ATP} \\ \text{COA} \\ \text{NADH} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0. & 0. & 0. & -1. & 0. & 0. \\ 0. & -1. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & -1. & 0. \end{pmatrix} * \begin{pmatrix} \text{ACAL} \\ \text{ACCOA} \\ \text{ACP} \\ \text{ADP} \\ \text{NAD} \\ \text{PYR} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5. \\ 1. \\ 10. \end{pmatrix}$$

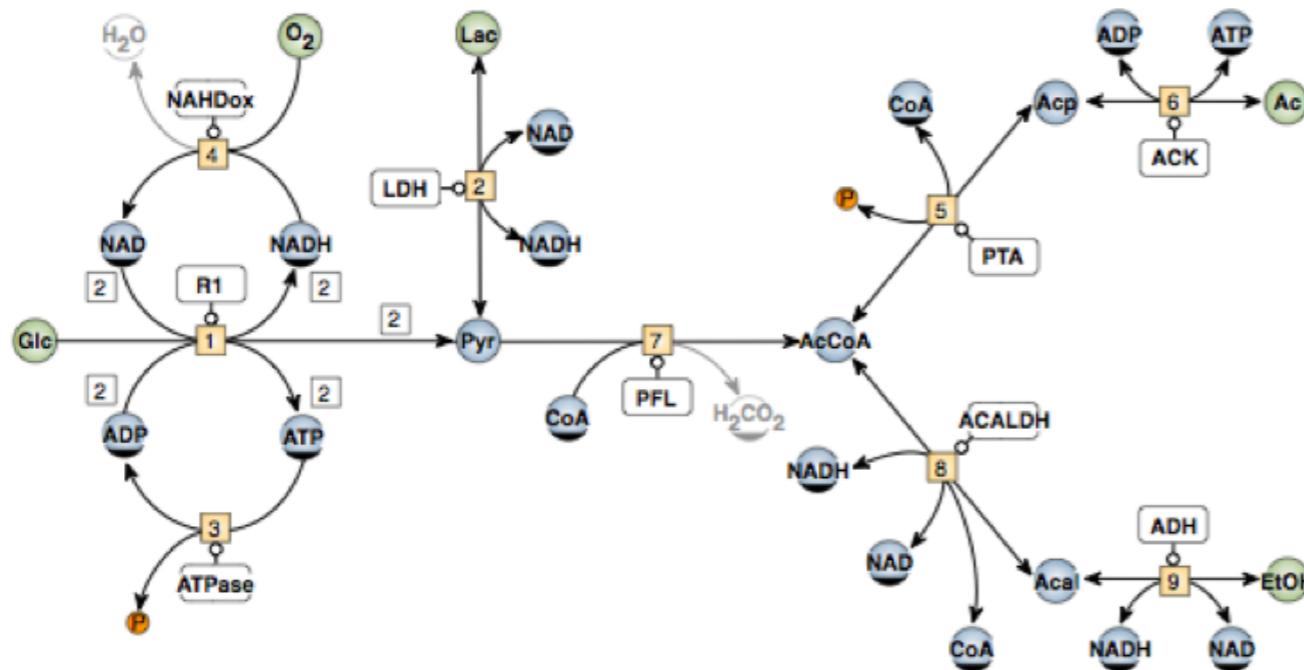
s vector of metabolite concentrations

L link matrix

si vector of independent metabolite concentrations

T vector of the sum of conserved moiety concentrations

Reaction network: st. st. constraint



$$\dot{Aca} = 0 = v_8 - v_9; v_8 = v_9$$

$$\dot{AcCoA} = 0 = -v_5 + v_7 - v_8; v_5 = v_7 - v_8 = v_7 - v_9$$

$$\dot{AcP} = 0 = v_5 - v_6; v_6 = v_5 = v_7 - v_9$$

$$\dot{ADP} = 0 = 2 \cdot v_1 + v_3 - v_6; 2 \cdot v_1 = v_3 - v_6 = v_3 - v_7 + v_9$$

$$\dot{ATP} = -\dot{ADP}$$

$$\dot{CoA} = -\dot{AcCoA}$$

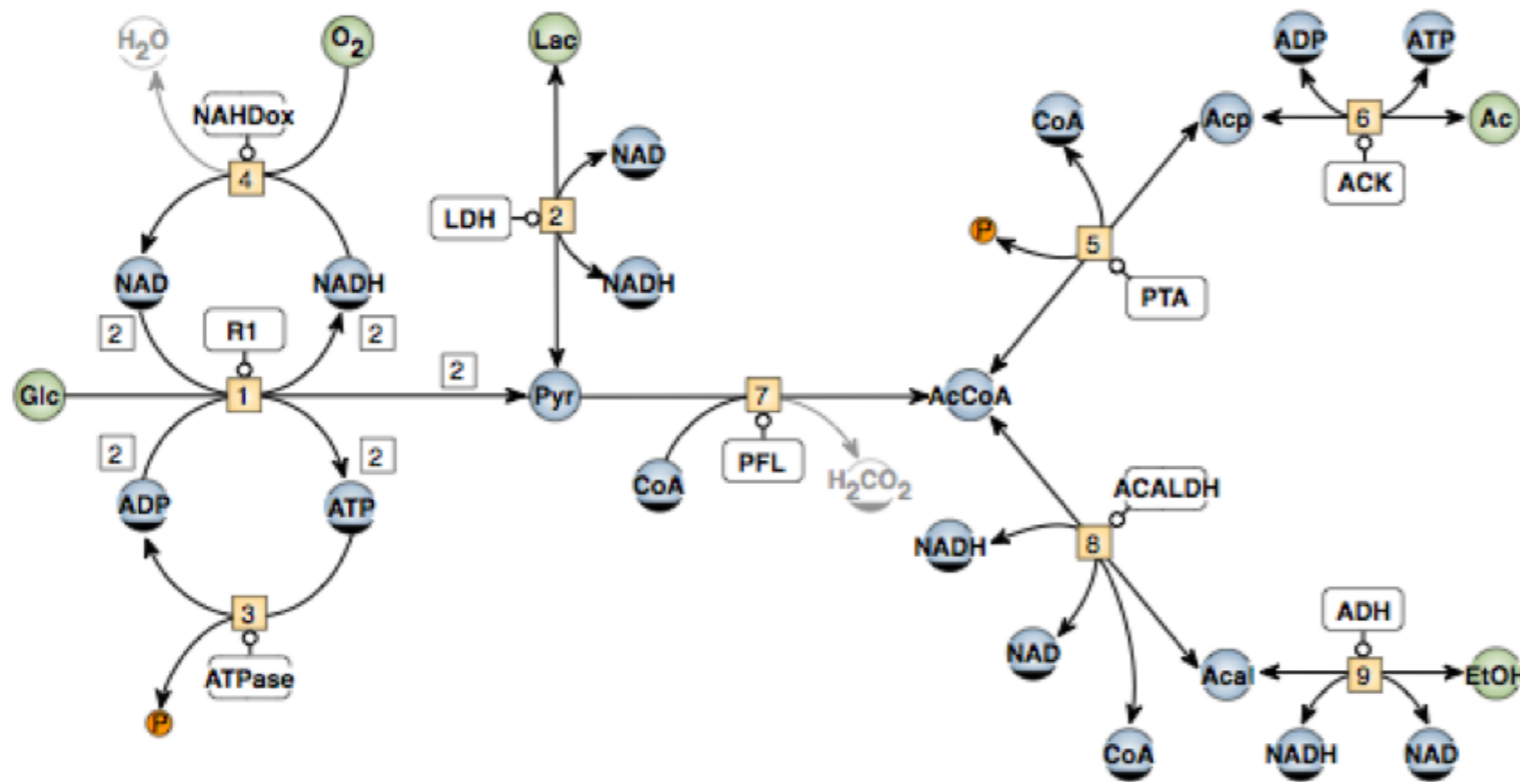
$$\dot{Pyr} = 0 = 2 \cdot v_1 - v_2 - v_7; v_2 = 2 \cdot v_1 - v_7 =$$

$$v_3 - v_7 + v_9 - v_7 = v_3 - 2 \cdot v_7 + v_9$$

$$\dot{NAD} = 0 = -v_1 + v_2 + v_4 + v_8 + v_9; v_4 = 2 \cdot v_1 - v_2 - v_8 - v_9 =$$

$$v_3 - v_7 + v_9 - v_2 - 2 \cdot v_9 = v_3 - v_7 - v_9 - v_2 = v_3 - v_7 - v_9 - v_3 + 2 \cdot v_7 - v_9 = v_7 - 2 \cdot v_9$$

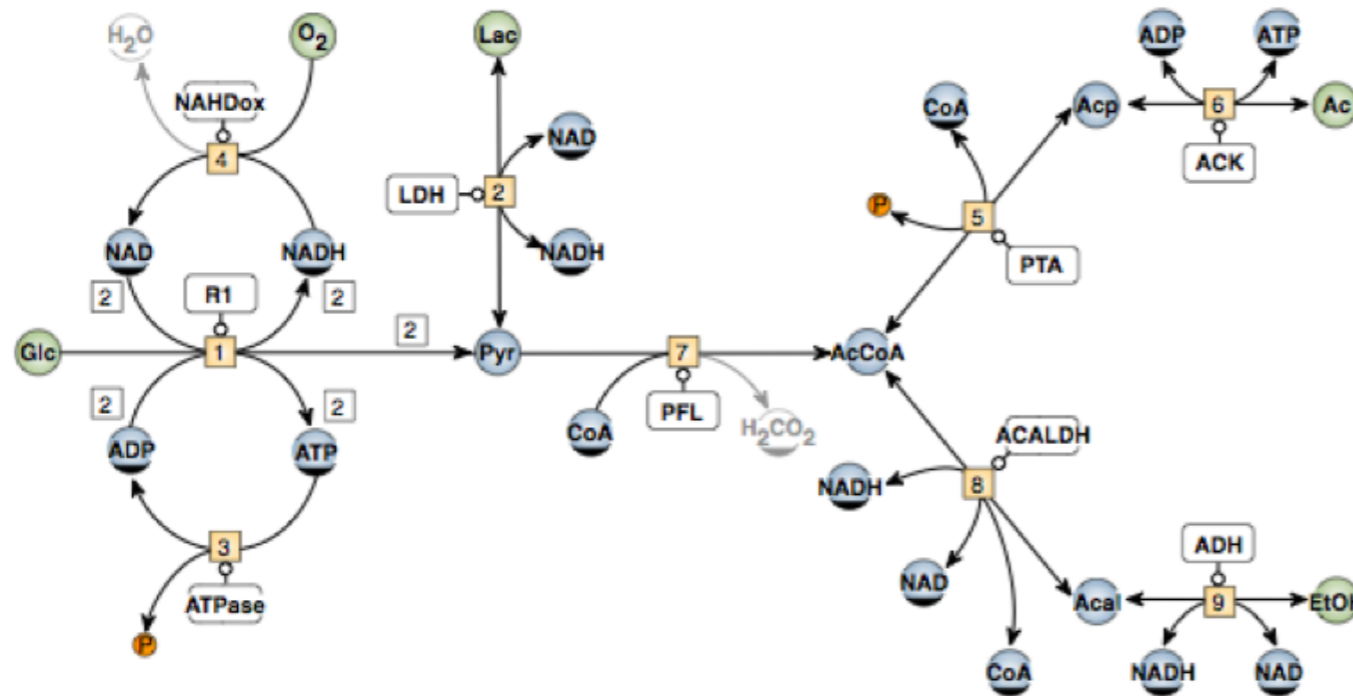
Reaction network: K matrix



$$\begin{matrix} \mathbf{J} \\ \left(\begin{array}{c} J3 \\ J7 \\ J9 \\ J1 \\ J2 \\ J4 \\ J5 \\ J6 \\ J8 \end{array} \right) \end{matrix} = \begin{matrix} \mathbf{K} \\ \left(\begin{array}{ccc} 1. & 0. & 0. \\ 0. & 1. & 0. \\ 0. & 0. & 1. \\ 0.5 & -0.5 & 0.5 \\ 1. & -2. & 1. \\ 0. & 1. & -2. \\ 0. & 1. & -1. \\ 0. & 1. & -1. \\ 0. & 0. & 1. \end{array} \right) \end{matrix} * \begin{matrix} \mathbf{Ji} \\ \left(\begin{array}{c} J3 \\ J7 \\ J9 \end{array} \right) \end{matrix}$$

- J** steady state reaction rate (flux) vector
- K** kernel, nullspace of stoichiometry matrix
- Ji** independent flux vector

Reaction network: K matrix; flux modes



$$\begin{matrix} \text{J} \\ \text{J3} \\ \text{J7} \\ \text{J9} \\ \text{J1} \\ \text{J2} \\ \text{J4} \\ \text{J5} \\ \text{J6} \\ \text{J8} \end{matrix} = \begin{pmatrix} 1. & 0. & 0. \\ 0. & 1. & 0. \\ 0. & 0. & 1. \\ 0.5 & -0.5 & 0.5 \\ 1. & -2. & 1. \\ 0. & 1. & -2. \\ 0. & 1. & -1. \\ 0. & 1. & -1. \\ 0. & 0. & 1. \end{pmatrix} * \begin{matrix} \text{Ji} \\ \text{J3} \\ \text{J7} \\ \text{J9} \end{matrix}$$

J steady state reaction rate (flux) vector

K kernel, nullspace of stoichiometry matrix

Ji independent flux vector

$J3 + 0.5 \cdot J1 + J2$ homolactic fermentation

$2 \cdot J3 + J1 + 2 \cdot J2 + J7 - 0.5 \cdot J1 - 2 \cdot J2 + J4 + J5 + J6$

$2 \cdot J3 + 0.5 \cdot J1 + J7 + J4 + J5 + J6$ homo acetate formation

$J9 + 0.5 \cdot J1 + J2 - 2 \cdot J4 - J5 - J6 + J8 + 2 \cdot (\text{column 2}) + 3 \cdot (\text{column 1})$

$3 \cdot J3 + J1 + 2 \cdot J7 + J5 + J6 + J8 + J9$ mixed acid fermentation

